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ON THE AVERAGE TRANSFER COEFFICIENT IN PERIODIC HEAT EXCHANGE—I

SOLID WITH NEGLIGIBLE THERMAL RESISTANCE

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Abstract—A simple analysis is presented for the one-dimensional regenerator with a boundary condition of the third kind and a stepwise change in gas temperature. The resulting storage capacity and extreme temperatures are affected by different heat-transfer coefficients during the heating and cooling periods as well as by the phase angle of heating. An analytical expression for the correct average transfer coefficient is developed and it is shown that the harmonic average recommended frequently can result in heat flux densities and matrix temperatures which are too high.

Although this analysis is exact only for the limiting case of zero matrix length or infinite heat capacity of the gas the result may also be applied to more realistic situations.

NOMENCLATURE

c,	specific heat of solid [$\mathrm{J\,kg^{-1}\,K^{-1}}$];
D,	thickness of parallel slab (diameter of cylinder or sphere) [m];
h,	heat-transfer coefficient [$\mathrm{Wm^{-2}\,K^{-1}}$];
k,	thermal conductivity of solid [$\mathrm{Wm^{-1}\,K^{-1}}$];
L,	characteristic length [m];
q,	heat-flux density [$\mathrm{Wm^{-2}}$];
t,	time [s];
T,	temperature [K];
α,	thermal diffusivity [$\mathrm{m^2\,s^{-1}}$];
ρ,	density of solid [$\mathrm{kg\,m^{-3}}$];
ϕ,	phase angle of heating [rad];
ω,	rotational frequency [$\mathrm{s^{-1}}$].

Dimensionless quantities

Bi,	$= \bar{h}/(\rho c\omega L) = h/(\rho c\omega L)$ Biot number;
Fo,	$= \sqrt{[\alpha/(\omega L^2)]}$ Fourier number;
H,	$= q/h(T_{g2} - T_{g1})$ heat-flux parameter;
Δ,	temperature defined by equation (15);
θ,	$= \dfrac{T - T_{g1}}{T_{g2} - T_{g1}}$, temperature;
τ,	$= \omega t$ time.

Subscripts

1,	for cooling time;
2,	for heating time;
g,	gas;
$g1, g2$,	see Fig. 1;
s,	solid;
ω,	per cycle.

1. INTRODUCTION

THE REGENERATIVE heat-transfer process is characterized by an alternating heat flux between a gas and a solid matrix and its performance depends largely on the mechanisms of conduction and convection. By comparison, the indirect heat exchanger or recuperator is adequately described by convection mechanisms alone. Consequently the thermal analysis of the regenerator is usually more complicated and has been the subject of continuing research and development.

Generally, the temperature of the regenerator matrix has to be studied in three dimensions: time, depth from the solid surface and length in the gas-flow direction. Numerous versions of this problem, with differing degrees of simplification, have been studied successfully but, to our knowledge, a general analytical result in closed form has still not been found. Solutions in the form of integral equations [1] and slowly converging series [2], and a numerical approach [3], are available but with none of these is the general influence of the system parameters on the regenerator performance shown very clearly. Based on the assumption of similarities between recuperators and regenerators [4] an extensive analytical study has been carried out [5]. This resulted in an equivalent overall heat-transfer coefficient for use in the total energy balance. The analysis is practicable for long regenerators with a stationary matrix, an arrangement frequently encountered in the steel industry. Then a number of simplifications can be introduced with almost no loss in accuracy. However, for shorter units such as the rotary regenerator the complete solution must be employed because the higher harmonics of the matrix temperature become important throughout the regenerator. In order to reduce the calculation effort a graphical method has been proposed as an alternative [5].

A similar but more idealized approach [6] leads to results in the form of design charts. In this case it is assumed that the gas temperatures are symmetric, i.e. the mean gas-to-gas temperature difference averaged over the whole period is the same throughout the regenerator. This of course limits the application to long regenerators and equal phase angles of heating and cooling. More recently a simpler analysis, more easily adaptable for use with computers, has been presented [7]. However, due to the assumption of a gas-solid boundary condition of the second kind the results are somewhat unrealistic in many practical situations. For the unidirectional regenerator, where the gas phase always enters from the same side, the method of Laplace transforms leads to a closed-form solution [8]. Although the analysis becomes much simpler as compared with the general problem, the solution is still complicated and therefore presented in several diagrams.

With most cryogenic and rotary regenerators the storage material is highly conductive and the assumption of a uniform temperature within a matrix element becomes justified. This essential simplification permits a closed-form solution for arbitrary gas-flow directions [9] which agrees well with numerical results for regenerator effectiveness [10]. Other effects like variable gas density [11] and arbitrary entering-gas temperature [12] have been investigated as well. Further, the commonly employed concept of constant heat-transfer coefficients along the flow path has been examined [13] with the conclusion that in the entrance region the matrix temperature may be quite different from the usually accepted values.

The rotary regenerator, although in its thermal behaviour not different from the stationary-matrix regenerator, has received particular attention over the past three decades because of its wide use as an efficient economizer in gas turbines, blast furnaces and power stations. Plant performance is often directly affected by regenerator performance so that a simple closed-form solution is essential in order to optimize the overall process. On the basis of physical models various approximate relations have been developed which seem to be useful for design purposes. Assuming that the gas phase is completely mixed at any cross-section of the regenerator one obtains a readily usable expression for the effectiveness [14]. Although this assumption appears unrealistic, matrix arrangements as in wire-screen or fast-rotating regenerators are approximated reasonably well. The former has been analyzed experimentally [15] and theoretically [16] on similar assumptions. Other models are aimed at optimizing the matrix length and rotational speed [17] or phase angle of heating [18] with respect to a maximum plant performance; some results for sample plants are presented in diagrams.

Finally, there is another type of heat exchanger in which at least part of the energy is exchanged by means of a regenerative process. In rotary kilns [19, 20] or coolers [21] as well as rotating driers or flakers [22, 23] the shell can be interpreted as the matrix. It undergoes periodic temperature changes while being contacted alternatively by a solid and a fluid phase. In all these cases periodicity was recognized but mostly it was not realized that one is dealing with a comparatively simple regenerator problem. The main feature distinguishing this type from the classical regenerator is that no axial coordinate needs to be considered for the periodic part (see [21]) because at any cross-section the gas and solid-phase temperatures are independent of time. On the other hand the phase times of heating and cooling as well as the relevant heat-transfer coefficients differ markedly from each other, an important fact of which the analysis has to take particular account.

It is mainly this last type of heat exchange which we have in mind in the present study although some general results applicable to the classical regenerator problem are discussed here and elsewhere [31]. With the assumption of time-independent gas temperatures simple but accurate relations can be derived for any values of individual phase times and heat-transfer coefficients. Examining existing theories and models for their direct applicability one concludes the following: the strictly analytical approach [8, 9, 11, 13] does not allow for a time-dependent heat-transfer coefficient because this would introduce a non-linear boundary condition whereby the chosen methods of solution become inapplicable. The theory of Hausen [5] makes provision for variable phase times and heat-transfer coefficients, particularly since he has introduced refinements of the basic method to account for temperature-dependent heat-transfer coefficients [24] and time-dependent gas-flow rates [25]. However, the concept of a harmonic average of the transfer coefficient, also adopted elsewhere [14], does not apply to the limiting case of constant gas temperatures during heating or cooling. Then the anticipated similarity between recuperators and regenerators vanishes and a different approach yields more accurate results. Another method [7] which might be considered here suffers from the slightly unrealistic boundary condition, discussed before. Obviously a numerical approach can always incorporate the desired features but no general results have been presented so far.

In conclusion, we find it necessary to develop new expressions for the average heat-transfer coefficient which apply to the class of problems described. The present paper deals with the case of infinite thermal conductivity of the matrix whereas the case of finite conductivity is investigated elsewhere [31]. In addition, a general analysis of periodic conduction with a step change in gas temperature is presented. With the step length as a parameter the results are directly applicable to problems of practical interest; in many cases they will represent a better approximation than existing results which are based on a sinusoidal change in gas temperature [26–28].

2. PARAMETERS AND ASSUMPTIONS

The temperature distribution in a solid body which is periodically heated and cooled reaches a pseudo-steady state and one is usually interested in one or more of the following pieces of information:

(a) Extreme temperatures in the solid;
(b) Amplitude of temperature oscillation in the symmetry axis;
(c) Storage capacity of a solid-volume element.

In dimensionless form these are described primarily by the two well-known parameters

$$Bi = \frac{hL}{k} \tag{1}$$

and

$$Fo = \frac{\sqrt{(\alpha/\omega)}}{L}, \tag{2}$$

where L is the characteristic length of the body and is equal to the solid volume divided by the surface area available for heat transfer. Hence

$$L = \frac{D}{n}, \qquad n = \begin{Bmatrix} 2 \text{ slab} \\ 4 \text{ cylinder} \\ 6 \text{ sphere} \end{Bmatrix}. \tag{3}$$

From a study of the general case of a finite body with finite thermal conductivity k there result conditions (item b) for the validity of simplified analyses; in these either a negligible thermal resistance or an infinite solid thickness is assumed. The number of potential applications may justify a separate treatment of these two cases, particularly as effects like the ones discussed here can be analyzed more generally.

In principle we have to consider a further parameter which depends on the nature of the gas-temperature oscillation. With a sinusoidal distribution the phase-shift parameter accounts for the time lag between gas and wall temperature whereas in the present analysis the step length of heating is an important quantity. However, it will be shown that under certain conditions the latter can be incorporated in the heat-transfer coefficient defined as an average over the complete cycle. Apparently, no phase-shift parameter needs to be considered here; with a stepwise change in gas temperature there is no time lag between gas and solid-surface temperature because as soon as the gas temperature changes the driving temperature difference between gas and solid must change its sign. This conclusion holds as long as any source term or constant temperature gradient can be separated from the periodic problem. Then the extreme temperatures will always be found on the surface and the results (items a and c) become comparatively simple.

In the present study the thermal conductivity of the solid must disappear and this is achieved by the following combination of equations (1) and (2):

$$Bi_{k\to\infty} = Bi \cdot Fo^2 = h/(\rho c \omega L). \tag{4}$$

Equation (4) can in fact be interpreted as the Biot number for this specific situation because it relates the energy transferred between gas and surface to the change in internal energy of the solid; this is equivalent to the classical definition. We prefer not to include any constants in this parameter which has to be kept in mind when results from different sources are compared. In the following analysis we delete the subscript in equation (4) but exclusively use this definition rather than equation (1).

For completeness we list the additional assumptions made throughout this study. The physical properties of the solid are independent of temperature and location, the individual heat-transfer coefficients do not vary over the periods of heating or cooling, the same boundary condition of the third kind is valid on all heat-transfer surfaces and finally, only the simplest geometries are considered, i.e. infinitely long cylinder and slab and sphere. More complex geometries like the finite cylinder and mixed boundary conditions have been analyzed [29] but on such a fundamental basis that the results are difficult to apply in practice.

3. ANALYSIS

Under the assumptions stated above the extreme temperatures of the solid and storage capacity result from a simple energy balance and the actual results can be obtained in different ways. In view of the general approach to be followed for the more complicated cases [31] we firstly present a solution which is based on a Fourier-series representation of the gas temperature and on one heat-transfer coefficient valid for the complete cycle. Thereafter an expression for the equivalent average heat-transfer coefficient is derived for use in the general solution.

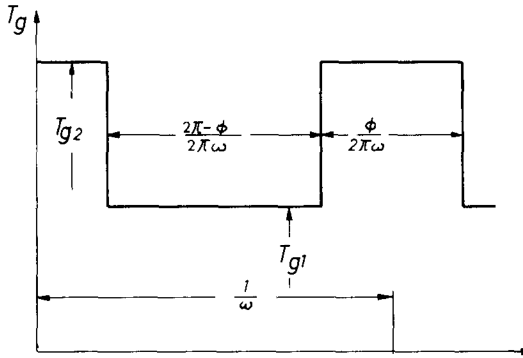

FIG. 1. Periodic gas-temperature distribution.

According to Fig. 1 the gas temperature as a continuous function of time becomes

$$\theta_g = \frac{T_g(t) - T_{g1}}{T_{g2} - T_{g1}} = \frac{\phi}{2\pi} + \frac{2}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\phi/2)}{n} \cos(2\pi n \omega t) \right\}. \tag{5}$$

With the dimensionless solid temperature

$$\theta_s = \frac{T_s(t) - T_{g1}}{T_{g2} - T_{g1}}$$

the energy balance for a solid element reads

$$\rho c L \frac{\mathrm{d}\theta_s}{\mathrm{d}t} = h(\theta_g - \theta_s) \tag{6}$$

which is rewritten to become

$$\frac{\mathrm{d}\theta_s}{\mathrm{d}\tau} = Bi(\theta_g - \theta_s). \tag{7}$$

This first-order ordinary differential equation is solved by direct integration and for our purposes the transient part of the solution can be disregarded. A more detailed account of the latter is given elsewhere [16]. In the periodic steady state we obtain for the solid temperature

$$\theta_s = \frac{\phi}{2\pi} + \frac{2}{\pi} \sum_{n=1}^{\infty} \times \left\{ \frac{\sin(n\phi/2)\cos(2\pi n\omega t) + (2\pi n/Bi)\sin(2\pi n \omega t)}{n \qquad 1 + (2\pi n/Bi)^2} \right\}. \tag{8}$$

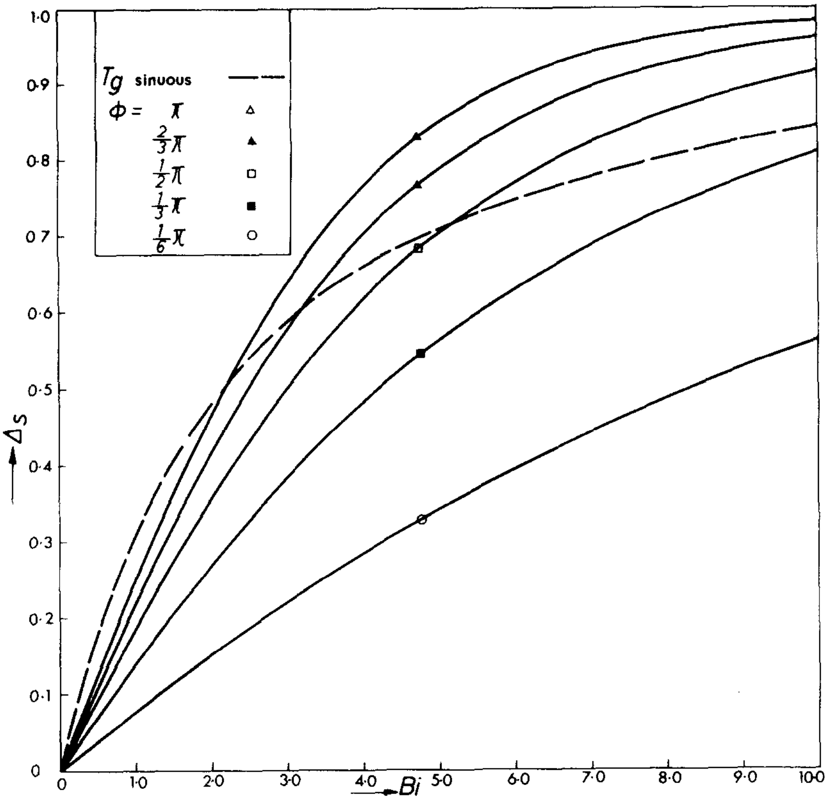

FIG. 2. Difference between maximum and minimum solid temperature for step change in gas temperature and with relative heating time as parameter.

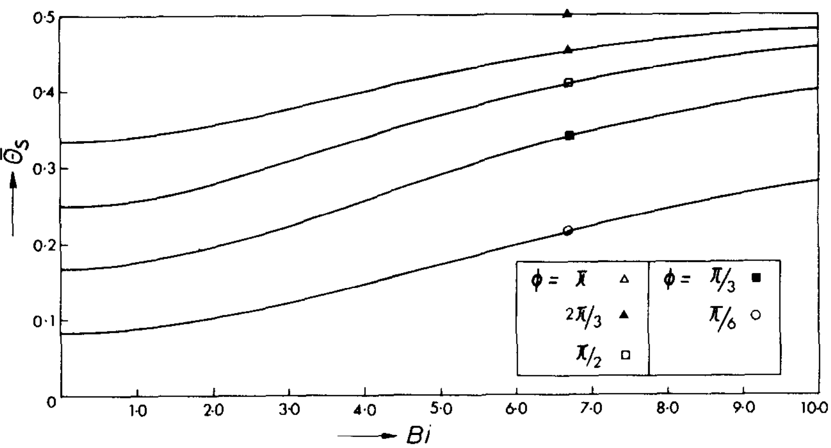

FIG. 3. Arithmetic average of the solid temperature; the curves are symmetric to $\bar{\theta}_s = 0.5$.

The time function in equation (8) could be transformed to

$$\frac{1}{\sqrt{[1+(2\pi n/Bi)^2]}}\cos[2\pi n\omega t - \tan^{-1}(2\pi n/Bi)]$$

as is usually done for similar cases [16, 30]. However, this would give the impression of a time lag between gas and solid-surface temperature which was shown not to exist. It is therefore preferred to leave the result in the form of equation (8). The difference between maximum and minimum solid temperature becomes

$$\theta_s|_{\phi/4\pi\omega} - \theta_s|_{4\pi-\phi/4\pi\omega} \equiv \Delta_s = \frac{8}{Bi}\sum_{n=1}^{\infty}\left\{\frac{\sin^2(n\phi/2)}{1+(2\pi n/Bi)^2}\right\}, \quad (9)$$

whereas the arithmetic average of the solid temperature is given by

$$(\theta_s|_{\phi/4\pi\omega} + \theta_s|_{4\pi-\phi/4\pi\omega})/2 \equiv \bar{\theta}_s = \frac{\phi}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}\left\{\frac{\sin(n\phi)}{n\left[1+\left(\frac{2\pi n}{Bi}\right)^2\right]}\right\} \quad (9a)$$

From these equations the extreme temperatures can readily be calculated and the results are shown in Figs. 2 and 3; it is noted that the temperatures are symmetric to $\phi = \pi$ and $\bar{\theta}_s = 0.5$ respectively.

The storage capacity is obtained in two different ways:

(a) Integration of the heat-flux density over the cooling time yields

$$q_\omega = h\int_{\phi/4\pi\omega}^{4\pi-\phi/4\pi\omega}[T_s(t)-T_{g1}]\,\mathrm{d}t = \frac{h(T_{g2}-T_{g1})}{\omega} \times\left\{\frac{\phi(2\pi-\phi)}{4\pi^2}-\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin^2(n\phi/2)}{n^2\left[1+\left(\frac{2\pi n}{Bi}\right)^2\right]}\right\}. \quad (10)$$

This is multiplied by the oscillation frequency and made dimensionless to give the heat flux parameter

$$H \equiv \frac{q}{h(T_{g2}-T_{g1})} = \frac{\phi(2\pi-\phi)}{4\pi^2} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin^2(n\phi/2)}{n^2\left(1+\frac{4\pi^2n^2}{Bi^2}\right)} \quad (11)$$

(b) Alternatively the result is obtained directly:

$$q_\omega = Lc\rho\Delta_s(T_{g2}-T_{g1}) \quad \text{or} \quad (12a)$$

$$H = 8\sum_{n=1}^{\infty}\left(\frac{\sin^2(n\phi/2)}{Bi^2+4\pi^2n^2}\right). \quad (12)$$

The results are illustrated in Fig. 4 from which the storage capacity is easily evaluated at any values of the gas temperatures, step length ϕ or other relevant parameters.

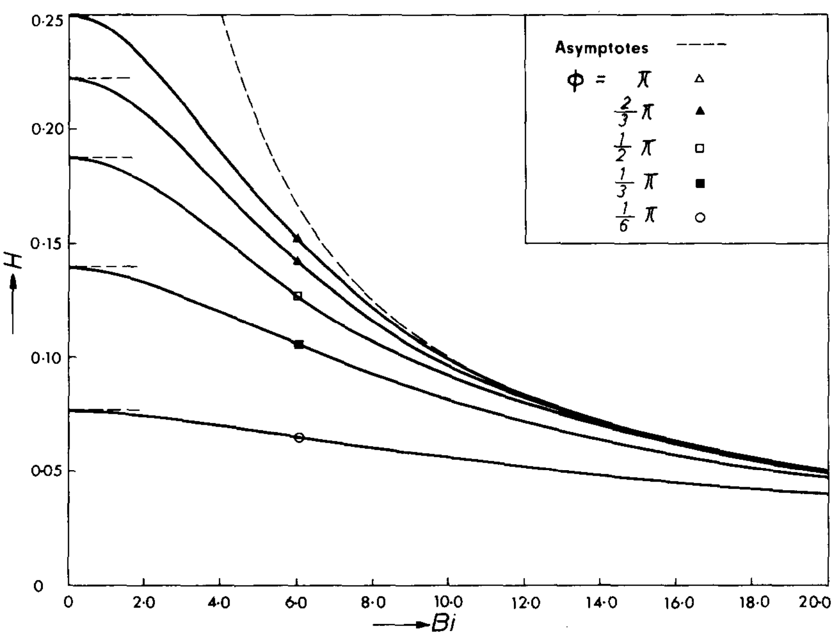

FIG. 4. Dimensionless storage capacity of a solid element at various relative heating times.

4. AVERAGE HEAT-TRANSFER COEFFICIENT

The previous analysis is general as far as different heating and cooling periods are concerned and is applicable to any gas-temperature distribution which can be represented by a Fourier series; the step change was chosen for its simplicity and direct use. The heat-transfer coefficient was taken as constant throughout the cycle or else the differential equation (7) becomes non-linear and the solution is complicated. However, if we restrict ourselves now to a step change in gas temperature and heat-transfer coefficient, a situation of practical interest, an even simpler analysis yields the exact average transfer coefficient for use in equations (9) and (12).

With a separate energy balance for both the heating and cooling time, i.e.

$$\left.\begin{aligned}\rho cL\frac{\mathrm{d}T_s}{\mathrm{d}t} &= h_2(T_{g2}-T_s)\ldots\frac{n}{\omega}\leqslant t\leqslant\frac{n+\phi/2\pi}{\omega}\\ \text{and}&\\ \rho cL\frac{\mathrm{d}T_s}{\mathrm{d}t} &= h_1(T_{g1}-T_s)\ldots\frac{n+\phi/2\pi}{\omega}\leqslant t\leqslant\frac{n+1}{\omega}\end{aligned}\right\} \quad (13)$$

one obtains, after integrating and making use of the "switching condition" the following:

$$\left.\begin{aligned}\frac{T_{s,\max}-T_{g2}}{T_{s,\min}-T_{g2}} &= \exp(-Bi_2)\\ \text{and}&\\ \frac{T_{s,\min}-T_{g1}}{T_{s,\max}-T_{g1}} &= \exp(-Bi_1).\end{aligned}\right\} \quad (14)$$

These are two equations in two unknowns, $T_{s,\max}$ and $T_{s,\min}$, and after some arithmetic we find that

$$\frac{T_{s,\max}-T_{s,\min}}{T_{g2}-T_{g1}} \equiv \Delta_s = \frac{1}{\dfrac{1}{\exp(Bi_1)-1}+\dfrac{1}{1-\exp(-Bi_2)}} \quad (15)$$

and

$$\frac{\frac{T_{s,\max}+T_{s,\min}}{2}-T_{g1}}{T_{g2}-T_{g1}} \equiv \bar{\theta}_s = 0.5\frac{(\exp Bi_1+1)(\exp Bi_2-1)}{\exp(Bi_1+Bi_2)-1}. \quad (15a)$$

The individual Biot parameters incorporate both the heat-transfer coefficient and the phase time. From equation (13) and (14) it follows that

$$Bi_1 = \frac{h_1(2\pi-\phi)}{\rho c\omega L2\pi}; \quad Bi_2 = \frac{h_2\phi}{\rho c\omega L2\pi}. \quad (16)$$

We note further that with

$$h_1 = h_2 \quad \text{and} \quad \phi = \pi$$

one gets

$$Bi_1 = Bi_2 = Bi/2.$$

In this specific case equation (15) can be reduced to

$$\Delta_s = \frac{\exp(Bi_1)+\exp(-Bi_1)-2}{\exp(Bi_1)-\exp(-Bi_1)} = \tanh(Bi/4) \quad (17)$$

and

$$\bar{\theta}_s = 0.5.$$

In a different form this last result was already presented previously ([5], p. 338) where the limiting case of an infinitely short regenerator with a highly conductive matrix was investigated.

Apparently equation (17) is completely equivalent to equation (9) with $\phi = \pi$. Then the average heat-transfer coefficient to be applied in equations (9) and (12) is found by comparison of equations (15) and (17):

$$\bar{h} = 4\rho c\omega L \tanh^{-1} \frac{1}{\frac{1}{\exp(Bi_1)-1}+\frac{1}{1-\exp(-Bi_2)}}. \quad (18)$$

By using this transfer coefficient, which we may call the exponential average, the parameter ϕ becomes redundant and we only require the upper curves in Figs. 2 and 4. However, the graphs with $\phi < \pi$ are still useful for illustration of this parameter and for potential direct application. It is pointed out that according to equation (16) a change in the phase angle of heating has the same effect on the final result as the corresponding change in heat-transfer coefficient. Hence the lower curves of Figs. 2 and 4 as well as Fig. 3 apply also to differing individual transfer coefficients.

5. DISCUSSION OF RESULTS

From Fig. 4 it is seen that with increasing rotational frequency and all the other variables staying constant the storage capacity has a maximum at $\omega \to \infty$. This result may be somewhat surprising because simultaneously the amplitude of the temperature oscillation approaches zero (see Fig. 2) and one might have expected a maximum storage capacity at some finite value of ω. The asymptotic solution is found explicitly from equation (11). With $\phi = \pi$ and $Bi \to 0$ it follows that

$$H_{Bi\to 0} = 0.25, \quad (19)$$

a particularly simple result which can also be applied approximately to more realistic situations; the error is

$$\Delta H < +1\% \quad \text{for} \quad Bi \leqslant 0.7.$$

However, then we have to keep in mind our basic assumption of negligible thermal resistance; a large value of ω may impose limits on the maximum permissible value of L.

For very large Biot numbers another asymptotic solution is available. With $\Delta_s \to 1$, equation (12a) yields

$$H_{Bi\to\infty} = \frac{1}{Bi} \quad (20)$$

where

$$\Delta H < +1\% \quad \text{for} \quad Bi \geqslant 10.5.$$

Both asymptotic solutions are shown in Fig. 4 and may be usefully applied in certain ranges of the variables.

Further, we conclude from Fig. 4 that at low Biot numbers the influence of differing phase angles of heating and cooling or differing heat-transfer coefficients is very marked, whereas this effect becomes small at high Biot numbers. This is reasonable, since in the latter case the solid temperature approaches both gas temperatures such that the storage capacity of the solid is exhausted.

For $Bi_1 \neq Bi_2$ the larger of the two will have a smaller effect on the average than is expressed in the harmonic average recommended elsewhere [5, 14]. In the limit of one Biot number approaching zero or infinity both averages yield the same general result that the mean Biot number becomes zero and stays finite respectively. It is seen that these necessary conditions are not satisfied by the arithmetic average which was also recommended occasionally (see [14]); therefore the latter is not considered in the further discussion. In the notation chosen here the harmonic average becomes

$$\bar{h} = \frac{2}{\frac{\pi}{h_2\phi}+\frac{\pi}{h_1(2\pi-\phi)}}. \quad (21)$$

By comparison with equation (18) we realise that equation (21) (a) over-represents the influence of the higher heat-transfer coefficient on the average, particularly when both coefficients are large, and (b) does not take into account the absolute magnitude of the Biot number, i.e. the term ($\rho c\omega L$). From equation (18)

$$Bi/Bi_2 = f(Bi_1/Bi_2, Bi_1), \quad (22)$$

whereas from equation (21)

$$Bi/Bi_2 = f(Bi_1/Bi_2).$$

Equation (22) is illustrated in Fig. 5 and for comparison the harmonic average is shown as well. In certain ranges of the parameters the difference becomes

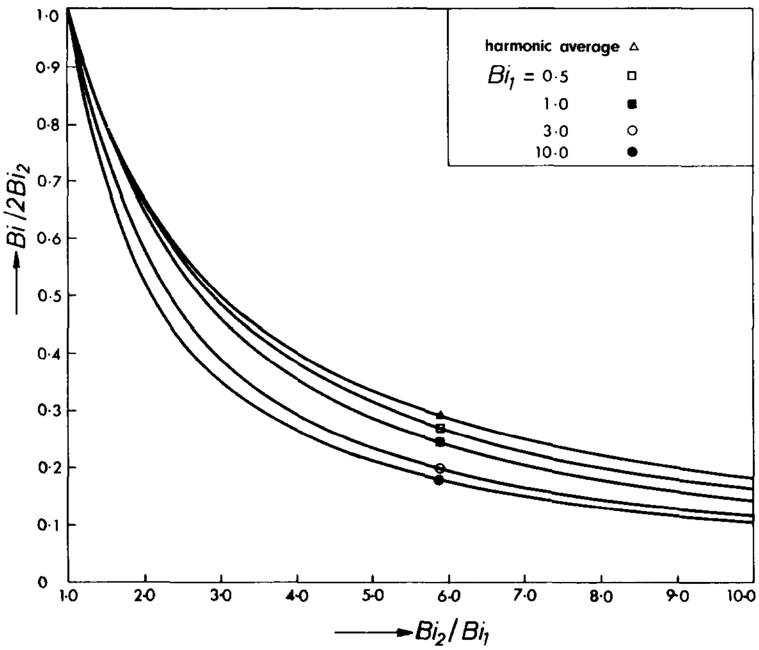

FIG. 5. Evaluation of the average Biot number from the individual parameters during cooling and heating.

substantial and can be explained in the following way:

The absorption or release of energy by the solid body is controlled by the gas film because we assumed a negligibly small internal resistance. Hence the total resistance between bulk fluid and solid is inversely proportional to the gas-to-surface temperature difference. According to equation (21) this is supposed to stay constant during heating or cooling and the sum of the two resistances yields the average over the complete cycle. However, with constant gas temperatures the driving force decreases exponentially with time so that the equivalent overall transfer coefficient is further reduced. In addition, the rate of temperature change in the solid depends on the value of $(\rho c \omega L)$ whence the difference between equations (18) and (21) must depend on the absolute values of the individual Biot numbers.

An estimate of the error involved in using equation (21) rather than equation (18) can be obtained from Fig. 6. This graph also facilitates the calculation of the true average transfer coefficient in that the exponential average can be evaluated without using tables of hyperbolic functions. Together with the upper curves of Figs. 2 and 4 one then determines the extreme temperatures in the solid as well as the storage capacity for any values of the independent variables. Although other combinations of the variables, resulting in different parameters, have been proposed [5, 28, 30], it is felt that with the ones presented here the relevant information on the system is obtained more directly. It is noted that in equation (18) the individual Biot parameters are interchangeable without any effect on the mean value; hence by exchanging indices Figs. 5 and 6 are applicable to cases where $Bi_2 < Bi_1$.

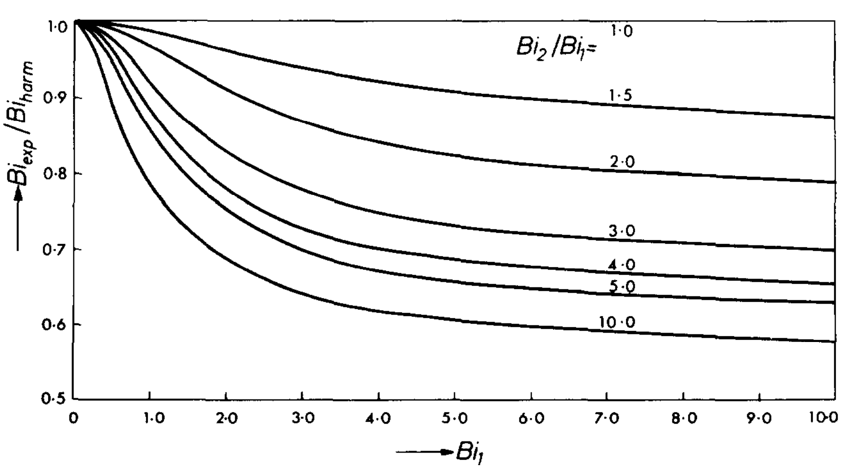

FIG. 6. Ratio of exponential to harmonic average of the Biot number for different combinations of the individual parameters.

6. APPLICATIONS

It is noticed in Fig. 2 that any error arising from the use of equation (21) rather than equation (18) is not projected linearly onto the extreme temperatures and storage capacity. At large Biot numbers the error in Δ_s or H becomes small and the harmonic average may be used with good approximation. On the other hand, at small Biot numbers the exponential functions in equation (18) can be approximated by the first two terms of their power-series representation and equations (18) and (21) become identical. Apparently, this is the reason why in the limit of infinite rotational frequency ω the harmonic average was found to be exact [14]. However in the range $0.5 < Bi < 5$ the error can be substantial; as an example the storage capacity of a copper wire of variable diameter was calculated from [5] and from the present equations and the results are shown in Table 1.

Table 1. Storage capacity of a copper plate at different conditions and error resulting from the use of the harmonic average of the transfer coefficient

Data: $\rho c = 3.44 \frac{MJ}{m^3 K}$; $\frac{1}{\omega} = 300$; $h_1 = 20$; $T_{g2} - T_{g1} = 200$

h_2	20		100		200	
$\phi/2\pi$	0.1667		0.5		0.75	
D	q_{exp}	$\frac{q_{harm}}{q_{exp}}$	q_{exp}	$\frac{q_{harm}}{q_{exp}}$	q_{exp}	$\frac{q_{harm}}{q_{exp}}$
0.0001	393	1.090	556	1.025	473	1.131
0.002	493	1.046	946	1.086	668	1.182
0.004	537	1.014	1 325	1.074	811	1.127
0.008	551	1.003	1 552	1.029	899	1.061

The analysis developed here is directly applicable to all situations where the gas temperatures stay constant (see Fig. 1). However, with many rotary and other regenerators the gas temperature at some interior point varies with time. A profile according to Fig. 7 is more likely to exist at some distance from the regenerator ends and one would like to know how accurately either of the discussed averages represents this distribution. For that purpose we repeated the calculations described in Section 4, but now with variable slopes of the gas temperatures. It was found that for $2 < Bi < 7$ and small slopes up to $(T_{g2} - T_{g1})/(10/\omega)$ the exponential average still supplies more accurate results than the harmonic one. With a very large slope, i.e. a fast change is gas temperature, even the harmonic average yields too little storage capacity; this might have been the reason for suggesting the use of an arithmetic average. However, as the choice of the slope was arbitrary no further details will be discussed here. The quantitative results are not significant as long as the slope is not linked to an energy balance. The reason for performing these calculations was to support the conclusion that with high heat capacities of the gas streams and/or short regenerators the exponential average is preferable to the harmonic one.

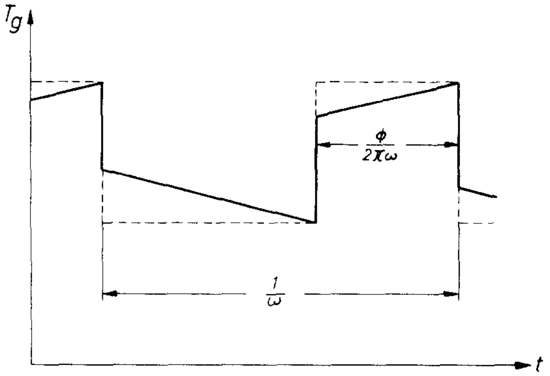

FIG. 7. Approximated gas-temperature distribution in a regenerator.

It may be noted that detailed analyses have been performed on the basis of Fig. 1 [14, 16] but on the assumption of an average transfer coefficient given by equation (21) [14]. This aforegoing seems to be inconsistent because if Fig. 1, apart from the absolute values of temperature, holds for any cross section then equation (18) also applies to the regenerator as a whole.

Finally, we may correct a statement made previously about adjusting the phase angles for unequal gas-flow rates in rotary (in particular wire-screen) regenerators. It was recommended [16] that the ratio of the cross-sectional areas of the matrix be made equal to the corresponding ratio of the mass-flow velocities of the gases so that the heat-transfer coefficients became the same for both streams. From equation (18) as well as equation (21) it is seen that this has no effect on the regenerator performance if the heat-transfer coefficient is linearly proportional to the velocity, the latter being inversely proportional to the phase angle. However, we know from experiments [32] that the fundamental results on cross flow over a wire [33] also hold for wire-mesh regenerators, hence over a wide range of velocities

$$h \sim (1/\phi)^{0.6}.$$

In that case the recommended design procedure may yield lower individual Biot numbers in comparison with the ones for $\phi = \pi$; according to equation (18) this results in a reduced efficiency of the exchanger. If, for purposes of reducing pressure losses, a change of the phase angles is still necessary its effect on the regenerator performance has to be analyzed by the use of equation (18) and Fig. 2. Particularly with gas-turbine plants the optimum phase angle for maximum efficiency must result from a balance of the effects of pressure drop and heat-exchanger performance.

With high-temperature regenerators the selection of the storage material depends on the maximum matrix temperature at the hot end of the regenerator. Here the exponential average can be used with good approximation because the gas temperature stays constant during the heating period. Apparently, the critical temperature as predicted by equation (18) is lower than the one from equation (21) so that less expensive materials can be used in certain cases.

7. CONCLUSIONS

Heat exchange between a solid and a gas of periodically varying temperature has been analyzed resulting in easily applicable equations for extreme matrix temperatures and storage capacity. However, the process was assumed to be "gas-film" controlled and the gas temperatures were supposed to stay constant. The main purpose was to derive an equation for the average heat-transfer coefficient, which accounts for variable times of heating and cooling and differing individual transfer coefficients. It was found that the exponential average as defined by equation (18) is exact whereas the harmonic average yields only approximate results, i.e. too high temperature and storage capacity of the matrix.

The possible use of the results in the design of rotary regenerators was discussed and it was concluded that the exponential average is applicable to short regenerators and/or processes with high heat capacities of the gas streams. The main advantage of this analysis over existing theories is that the effect of variable Biot numbers, given by equations (16), on the regenerator characteristics can be evaluated in a direct and simple way and therefore may be incorporated in an economical design procedure.

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SUR LE COEFFICIENT DE TRANSFERT MOYEN DANS UN ECHANGE PERIODIQUE DE CHALEUR: SOLIDE DE RESISTANCE THERMIQUE NEGLIGEABLE

Résumé—On présente une analyse simple de régénérateur unidimensionnel avec conditions aux limites de troisième espèce et changement discontinu de la température du gaz. La capacité d'accumulation et la valeur des températures extrêmes résultantes sont affectés par les coefficients de transfert thermique différents pendant les périodes de chauffage et de refroidissement ainsi que par le déphasage. On a développé une expression analytique donnant le coefficient moyen de transfert correct et on montre que la moyenne harmonique souvent recommandée peut conduire à une surestimation des densités de flux de chaleur et des températures de matrice.

Quoique cette étude ne soit exacte que pour le cas limite d'une matrice de longueur nulle ou d'un gaz de capacité calorifique infinie, les résultats sont également applicables à des situations plus proches de la réalité.

ÜBER DEN MITTLEREN WÄRMEÜBERGANGSKOEFFIZIENTEN BEI PERIODISCHEM WÄRMEAUSTAUSCH: UNENDLICH GUT LEITENDE SPEICHERMASSE

Zusammenfassung—Für den periodischen Wärmeaustausch zwischen einem Gas von sprungartig veränderlicher Temperatur und einem unendlich gut leitenden Material wird ein einfaches Berechnungsverfahren angegeben. Die Speicherkapazität und extremen Materialtemperaturen hängen nicht nur von der relativen Dauer der Heizzeit sondern auch von den unterschiedlichen Wärmeübergangskoeffizienten in Heiz-und Kühlzeit ab. Eine exakte Beziehung für den mittleren Wärmeübergangskoeffizienten wird hergeleitet, und es zeigt sich, daß der oft empfohlene harmonische Mittelwert zu hohe Werte für die Speicherkapazität und Matrixtemperatur liefert.

Obwohl das Verfahren streng genommen nur auf den unendlich kurzen Regenerator oder bei unendlicher Wärmekapazität der Gase anwendbar ist, lassen sich auch realistischere Falle mit guter Näherung behandeln.

СРЕДНИЙ КОЭФФИЦИЕНТ ПЕРЕНОСА ТЕПЛА ДЛЯ ТВЁРДОГО ТЕЛА С НЕЗНАЧИТЕЛЬНЫМ ТЕПЛОВЫМ СОПРОТИВЛЕНИЕМ ПРИ ПЕРИОДИЧЕСКОМ ИЗМЕНЕНИИ ТЕМПЕРАТУРЫ

Аннотация — Проведен анализ одномерного регенератора с граничными условиями третьего рода и ступенчатым изменением температуры газа. Результирующая аккумулирующая ёмкость и экстремальные температуры зависят от различных коэффициентов теплообмена при нагреве и охлаждении, а также от угла сдвига фаз при нагреве. Получено аналитическое выражение для соответствующего среднего коэффициента переноса и показано, что часто используемая средняя гармоника может дать очень высокие значения плотности теплового потока и температуры матрицы. Несмотря на то, что этот анализ точен только для предельного случая нулевой длины матрицы или бесконечной теплоемкости газа, полученные разультаты можно также использовать в более часто встречающихся в практике случаях.